

Algorithms

OBJECTIVES

After reading this chapter, the reader should be able to:

- Understand the concept of an algorithm.
- Define and use the three constructs for developing algorithms: sequence, decision, and repetition.
- Understand and use three tools to represent algorithms: flowchart, pseudocode, and structure chart.
- Understand the concept of modularity and subalgorithms.
- List and comprehend common algorithms.

8.1

CONCEPT

Figure 8-1

Informal definition of an algorithm used in a computer

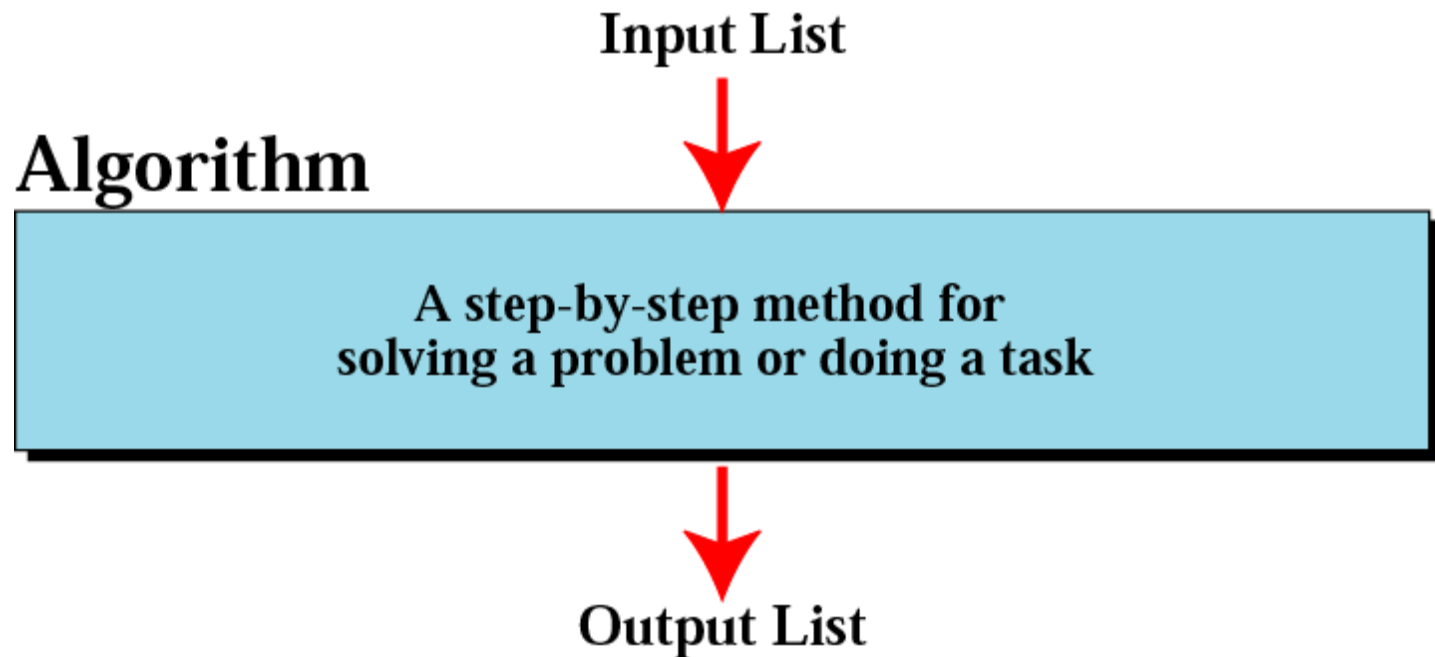


Figure 8-2

Finding the largest integer among five integers

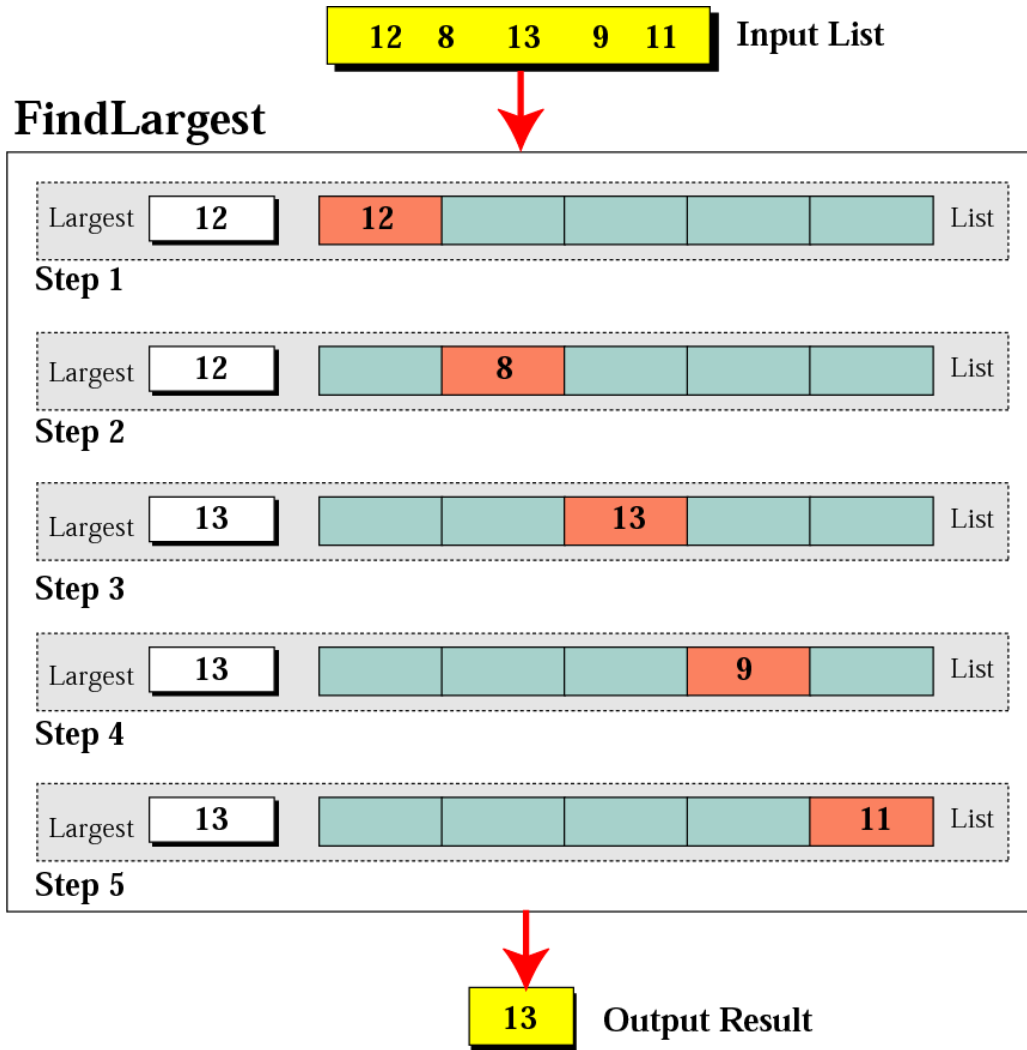


Figure 8-3

Defining actions in FindLargest algorithm

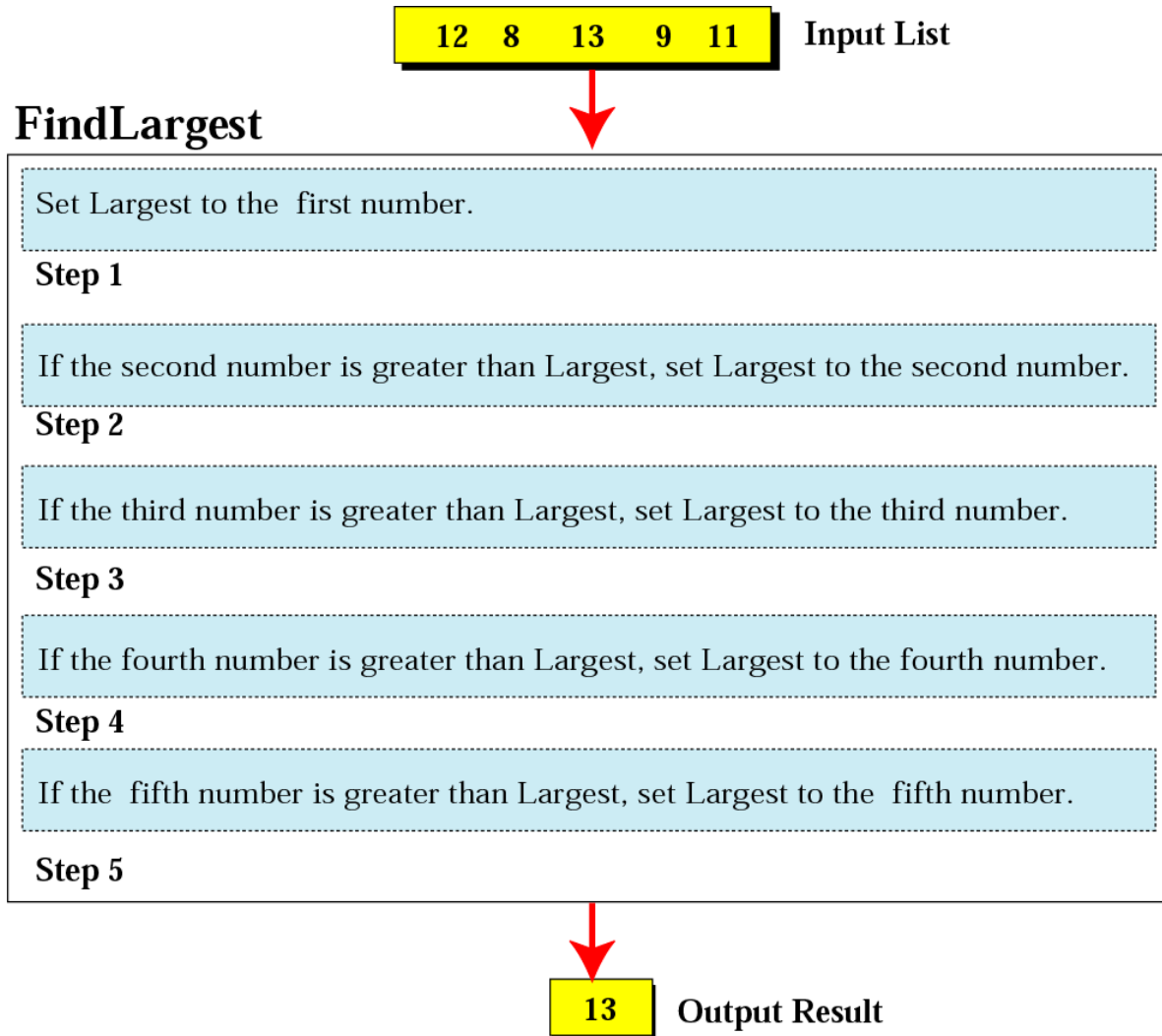


Figure 8-4

FindLargest refined

12 8 13 9 11 Input List

FindLargest

Set Largest to 0.

Step 0

If the current number is greater than Largest, set Largest to the current number.

Step 1

⋮

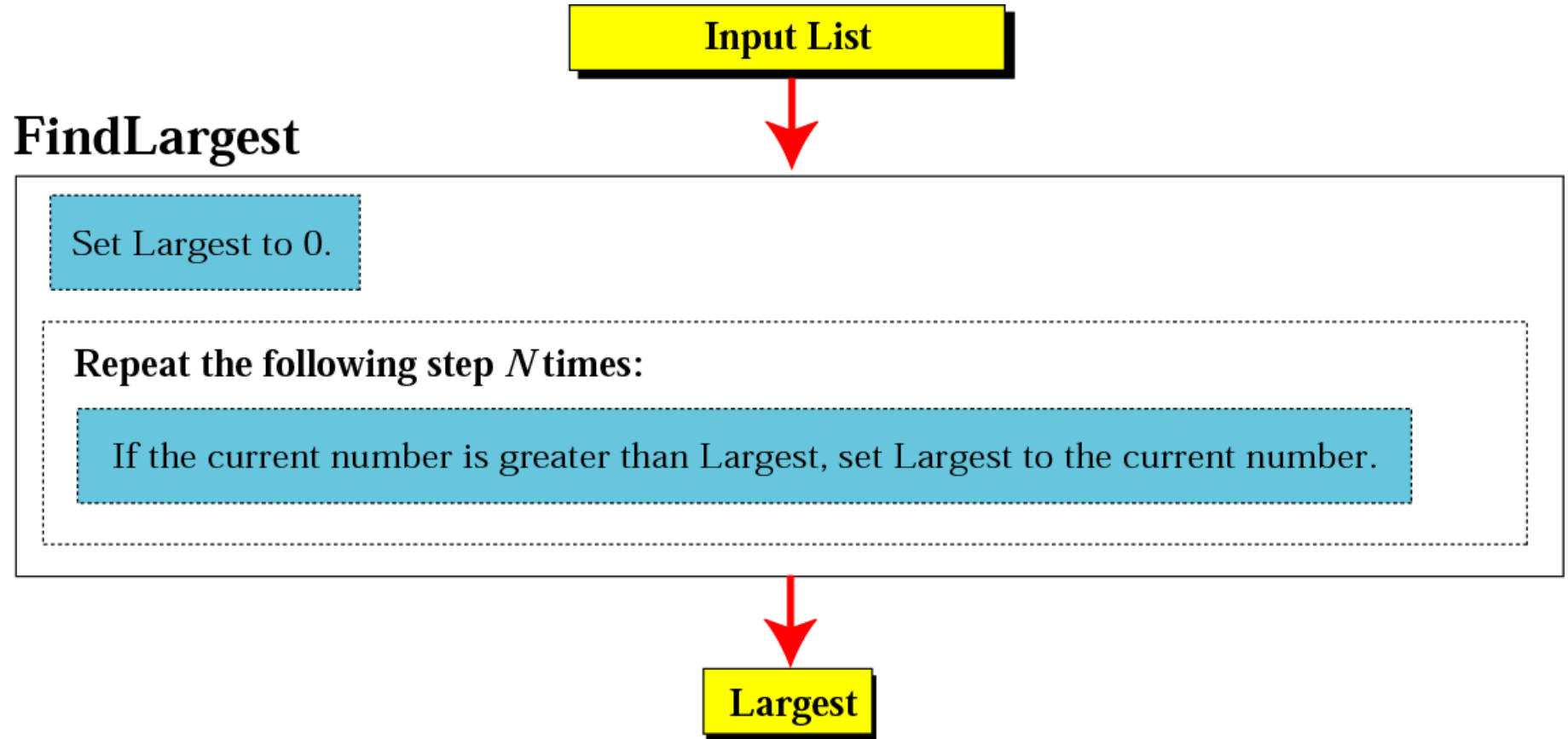
If the current number is greater than Largest, set Largest to the current number.

Step 5

13 Output Result

Figure 8-5

Generalization of FindLargest



8.2

THREE CONSTRUCTS

Figure 8-6

Three constructs

```
do action 1  
do action 2  
...  
...  
do action  $n$ 
```

a. Sequence

```
if a condition is true,  
then
```

```
do a series of actions
```

```
else
```

```
do another series of actions
```

b. Decision

```
while a condition is true,
```

```
do action 1  
do action 2  
...  
...  
do action  $n$ 
```

c. Repetition

8.3

ALGORITHM REPRESENTATION

Figure 8-7

Flowcharts for three constructs

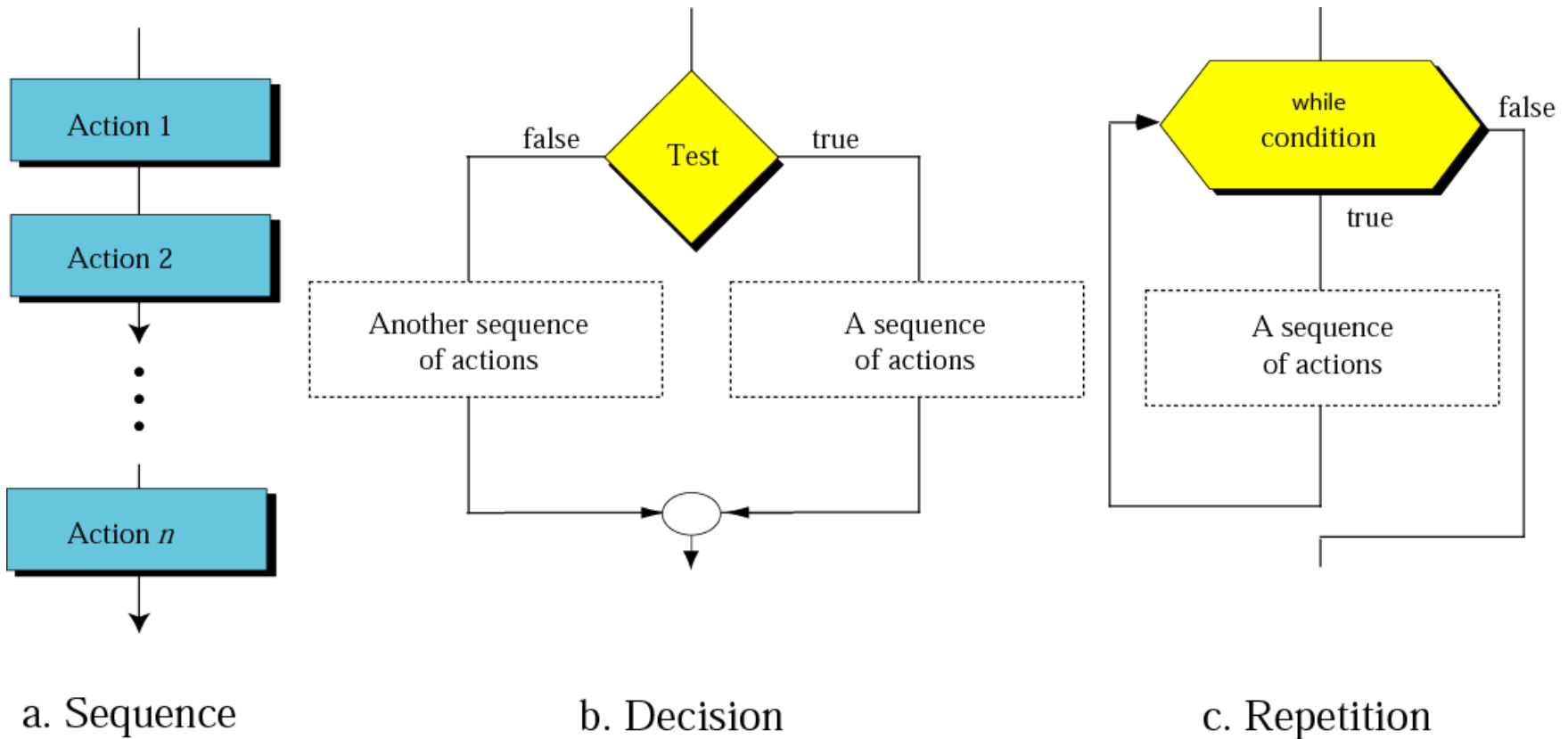
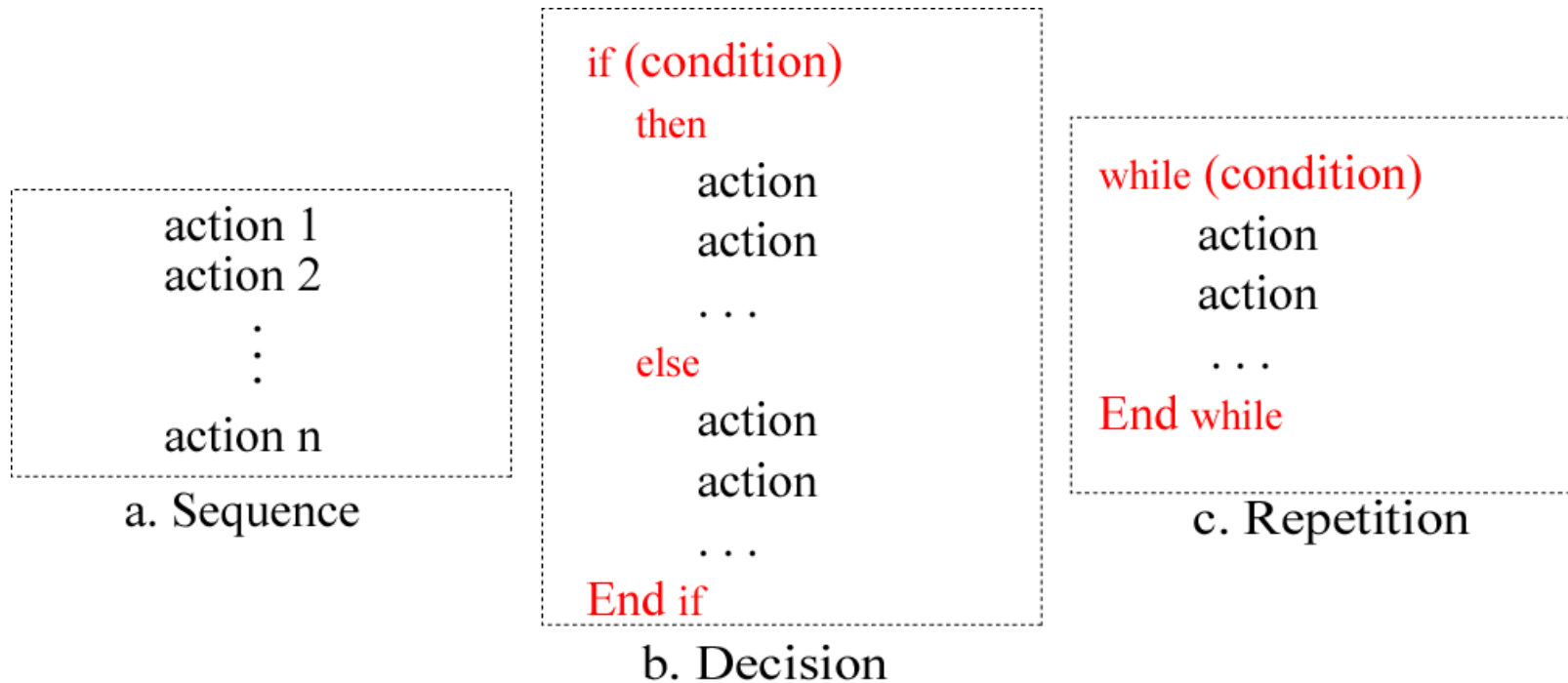


Figure 8-8

Pseudocode for three constructs



Example 1

Write an algorithm in pseudocode that finds the average of two numbers

Solution

See Algorithm 8.1 on the next slide.

Algorithm 8.1: Average of two

AverageOfTwo

Input: Two numbers

- 1. Add the two numbers**
- 2. Divide the result by 2**
- 3. Return the result by step 2**

End

Example 2

Write an algorithm to change a numeric grade to a pass/no pass grade.

Solution

See Algorithm 8.2 on the next slide.

Algorithm 8.2: Pass/no pass Grade

Pass/NoPassGrade

Input: One number

- 1. if (the number is greater than or equal to 70)
then**
 - 1.1 Set the grade to “pass”****else**
 - 1.2 Set the grade to “nopass”****End if**
- 2. Return the grade**

End

Example 3

Write an algorithm to change a numeric grade to a letter grade.

Solution

See Algorithm 8.3 on the next slide.

Algorithm 8.3: Letter grade

LetterGrade

Input: One number

- 1. if (the number is between 90 and 100, inclusive)
then**
 - 1.1 Set the grade to “A”****End if**
- 2. if (the number is between 80 and 89, inclusive)
then**
 - 2.1 Set the grade to “B”****End if**

Continues on the next slide

Algorithm 8.3: Letter grade (continued)

**3. if (the number is between 70 and 79, inclusive)
then**

3.1 Set the grade to “C”

End if

**4. if (the number is between 60 and 69, inclusive)
then**

4.1 Set the grade to “D”

End if

Continues on the next slide

Algorithm 8.3: Letter grade (continued)

5. If (the number is less than 60)

then

5.1 Set the grade to “F”

End if

6. Return the grade

End

Example 4

Write an algorithm to find the largest of a set of numbers. You do not know the number of numbers.

Solution

See Algorithm 8.4 on the next slide.

Algorithm 8.4: Find largest

FindLargest

Input: A list of positive integers

1. Set Largest to 0
 2. while (more integers)
 - 2.1 if (the integer is greater than Largest)
then
 - 2.1.1 Set largest to the value of the integer
 - End if
 - End while
 3. Return Largest
- End**

Example 5

Write an algorithm to find the largest of 1000 numbers.

Solution

See Algorithm 8.5 on the next slide.

Algorithm 8.5: *Find largest of 1000 numbers*

FindLargest

Input: 1000 positive integers

1. Set Largest to 0
 2. Set Counter to 0
 3. while (Counter less than 1000)
 - 3.1 if (the integer is greater than Largest)
 then
 - 3.1.1 Set Largest to the value of the integer
 - End if
 - 3.2 Increment Counter
 - End while
 4. Return Largest
- End**

8.4

MORE FORMA DEFINITION

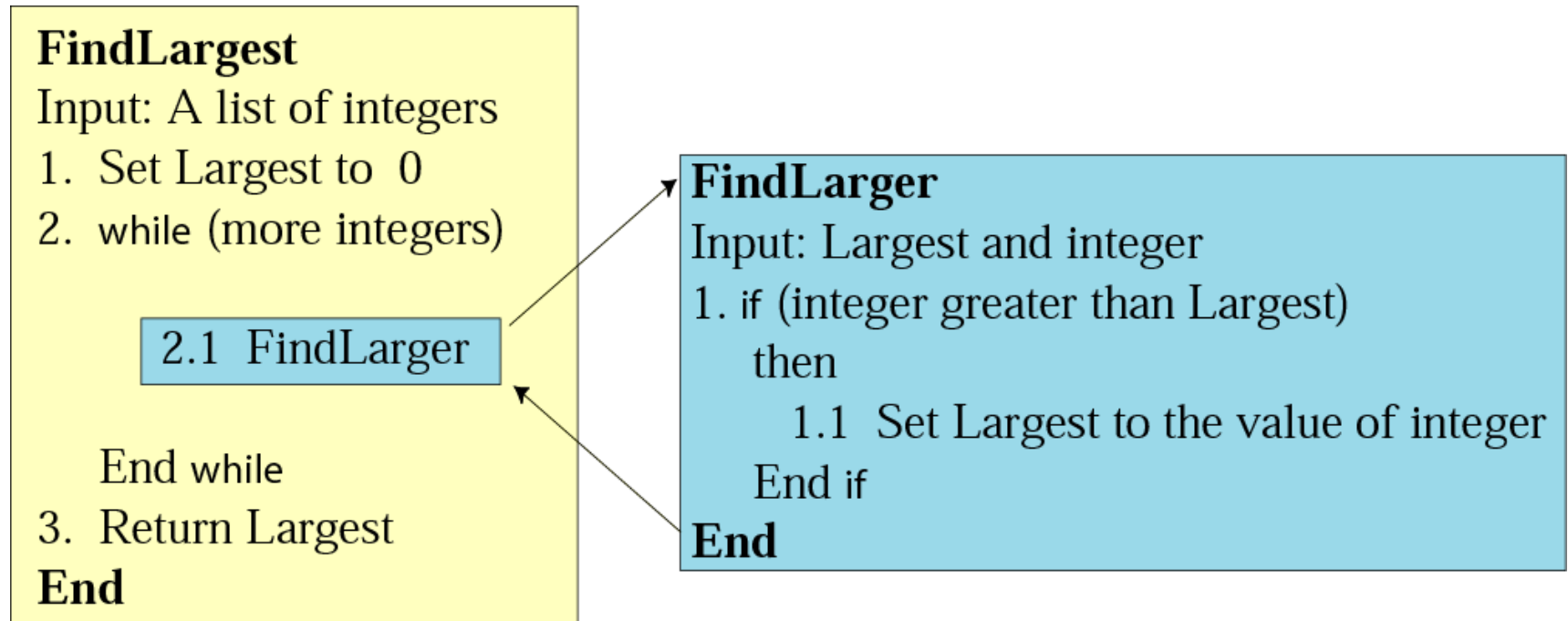
- ***Ordered set***
- ***Unambiguous steps***
- ***Effectiveness***
- ***Termination***

8.5

SUBALGORITHMS

Figure 8-9

Concept of a subalgorithm



Algorithm 8.6: Find largest

FindLargest

Input: A list of positive integers

1. Set Largest to 0
2. while (more integers)
 - 2.1 FindLarger
- End while
3. Return Largest

End

Subalgorithm: Find larger

FindLarger

Input: Largest and current integer

1. if (the integer is greater than Largest)
then
 - 1.1 Set Largest to the value of the integer
- End if**
End

8.6

BASIC ALGORITHMS

Figure 8-10

Summation

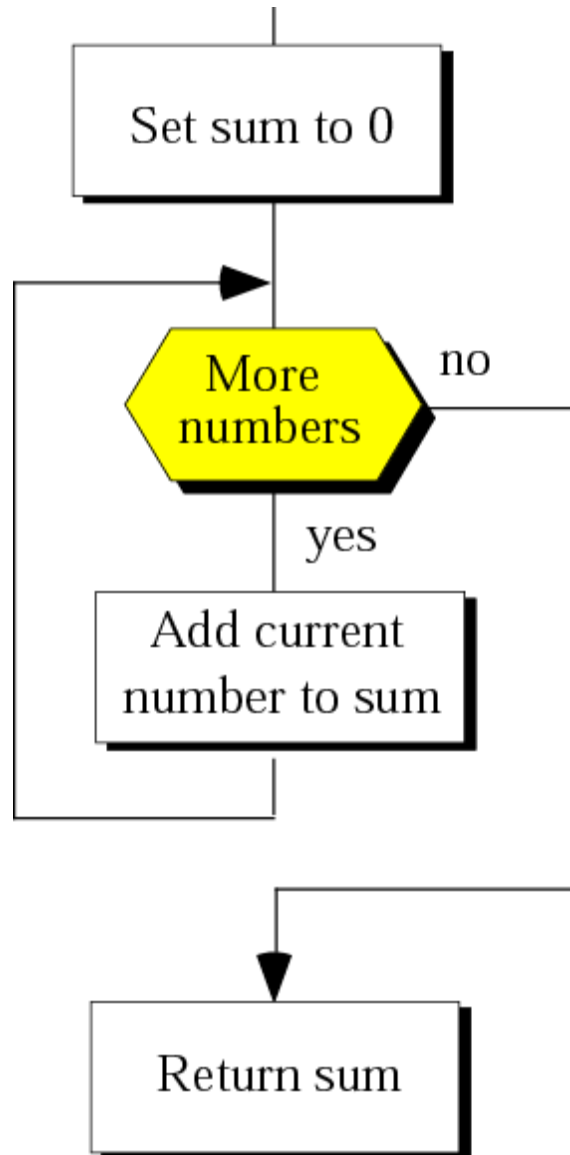


Figure 8-11

Product

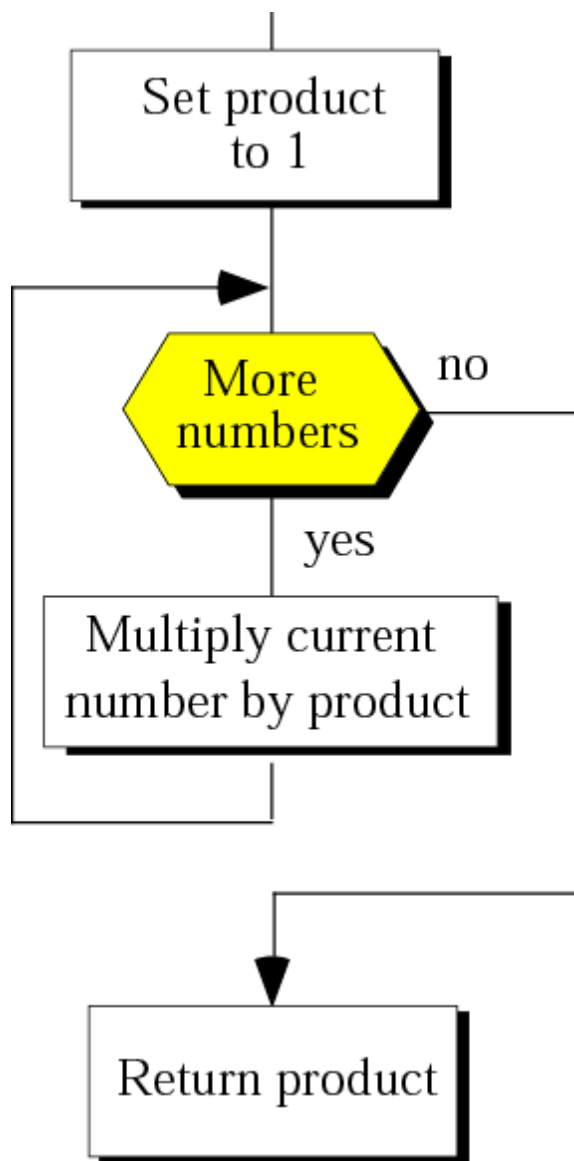


Figure 8-12

Selection sort

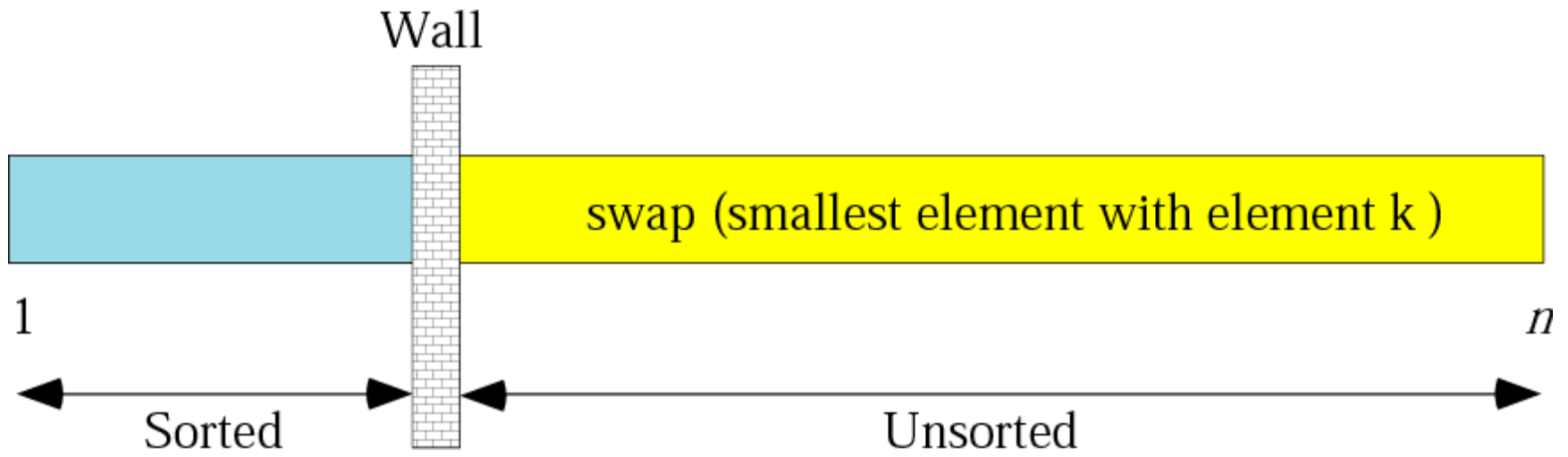
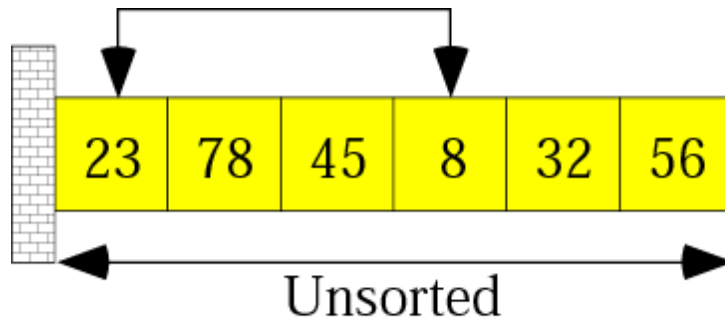
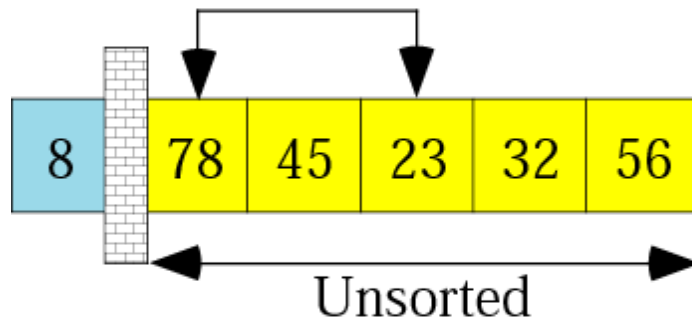


Figure 8-13: part I

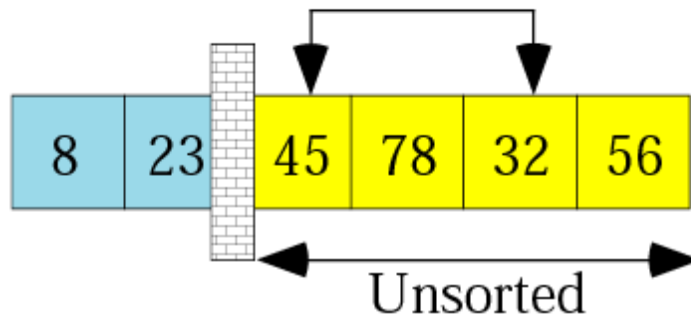
Example of selection sort



Original list



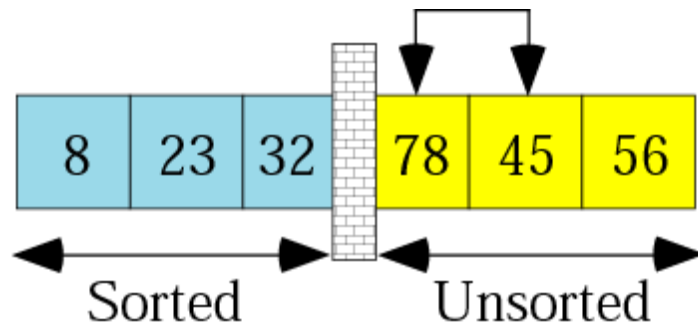
After pass 1



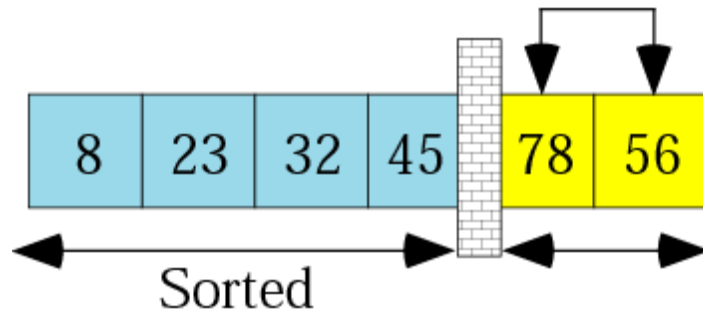
After pass 2

Figure 8-13: part II

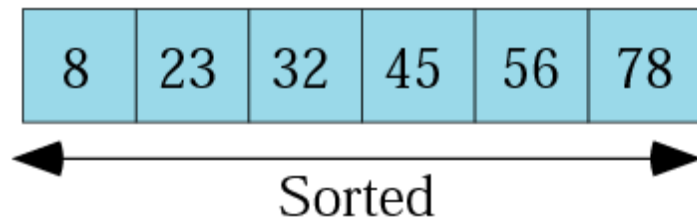
Example of selection sort



After pass 3



After pass 4



After pass 5

Figure 8-14

Selection sort algorithm

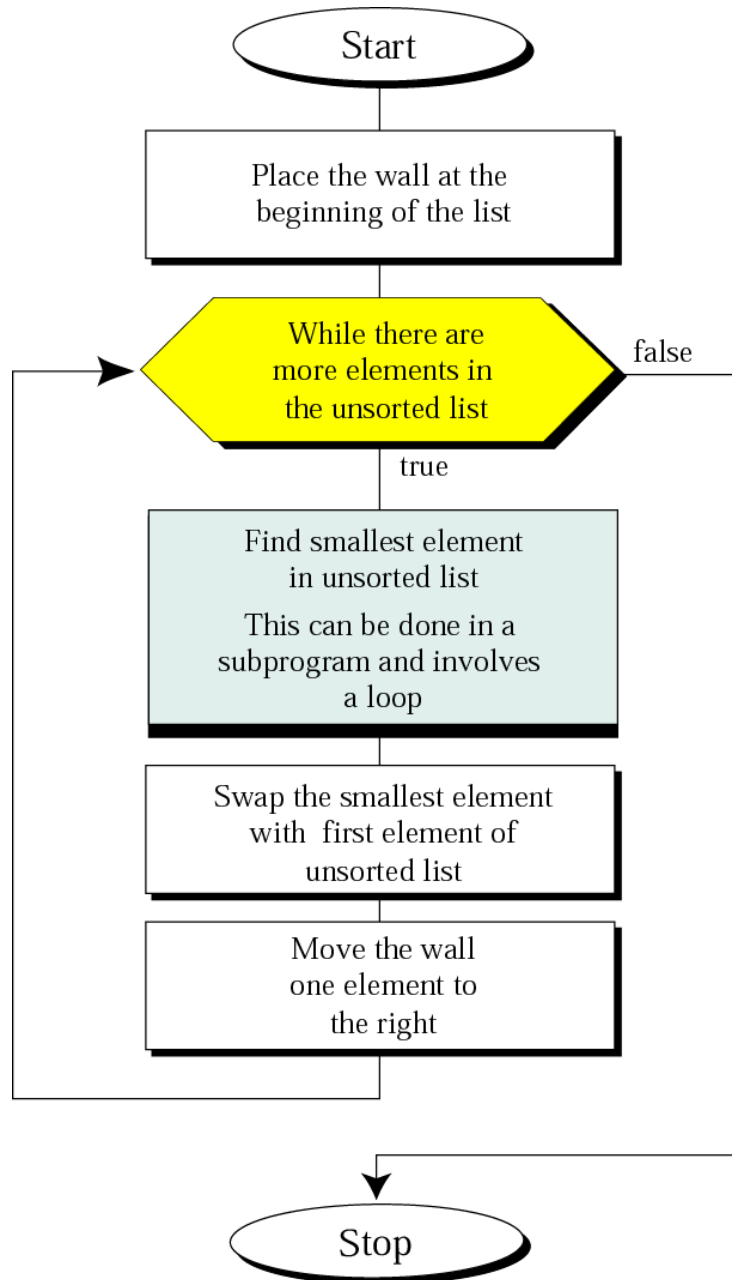


Figure 8-15

Bubble sort

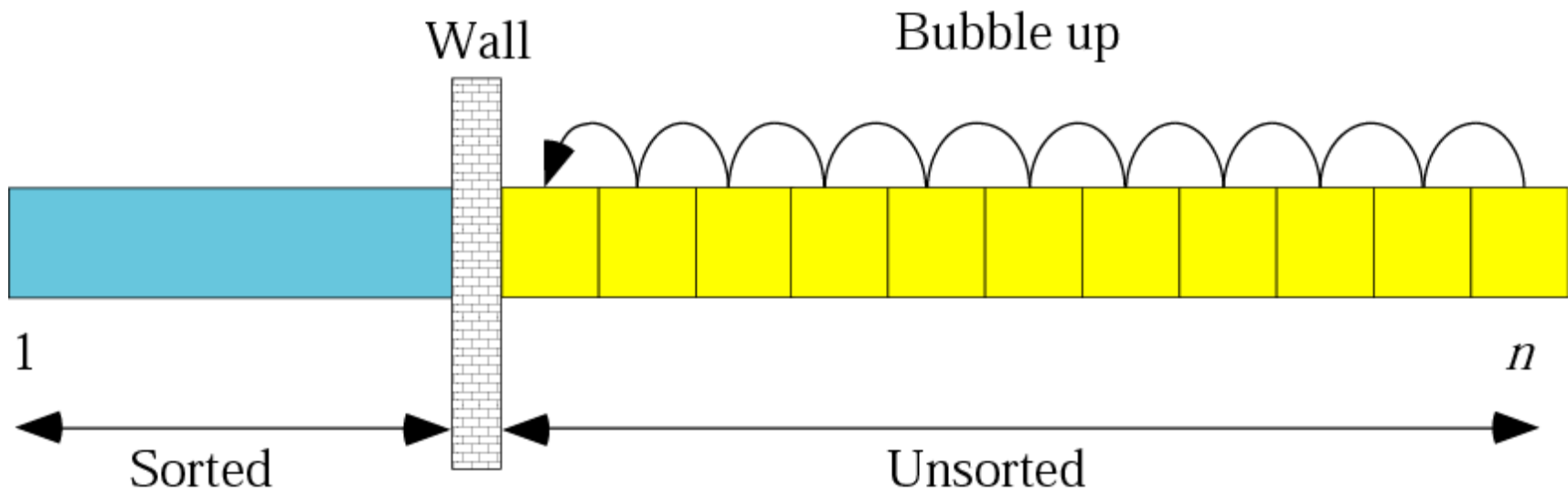
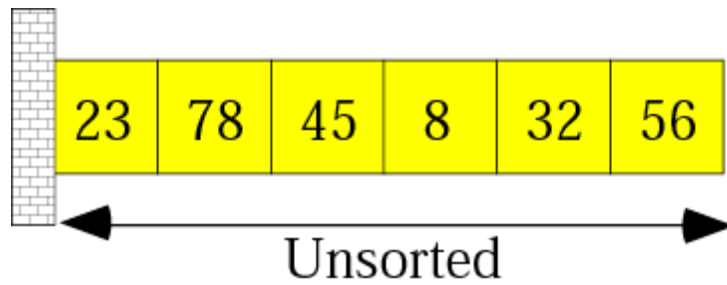
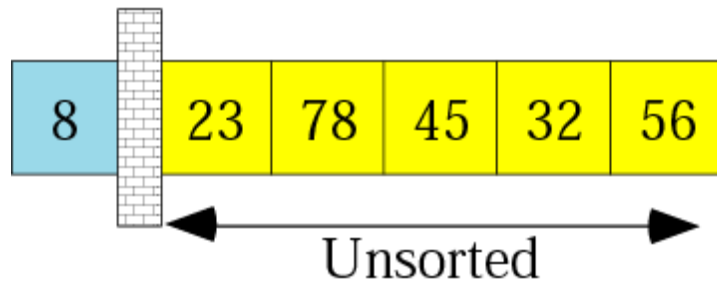


Figure 8-16: part I

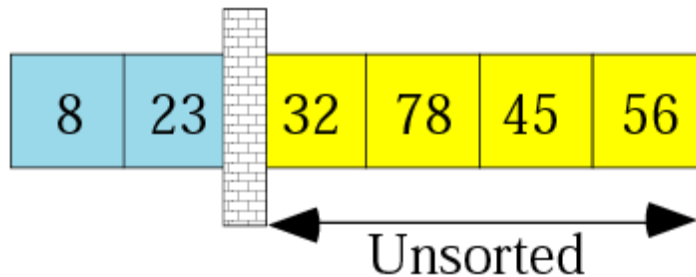
Example of bubble sort



Original list



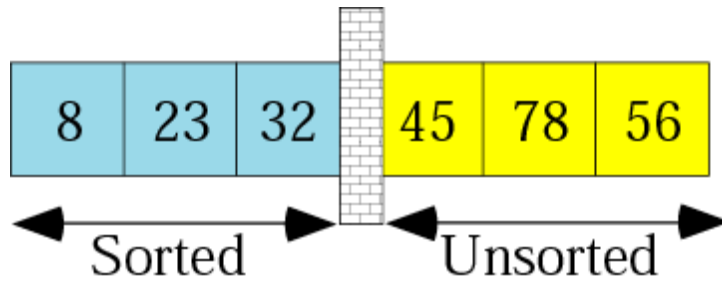
After pass 1



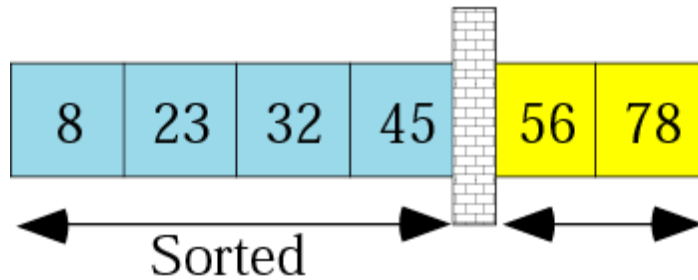
After pass 2

Figure 8-16: part II

Example of bubble sort



After pass 3



After pass 4
Sorted

Figure 8-17

Insertion sort

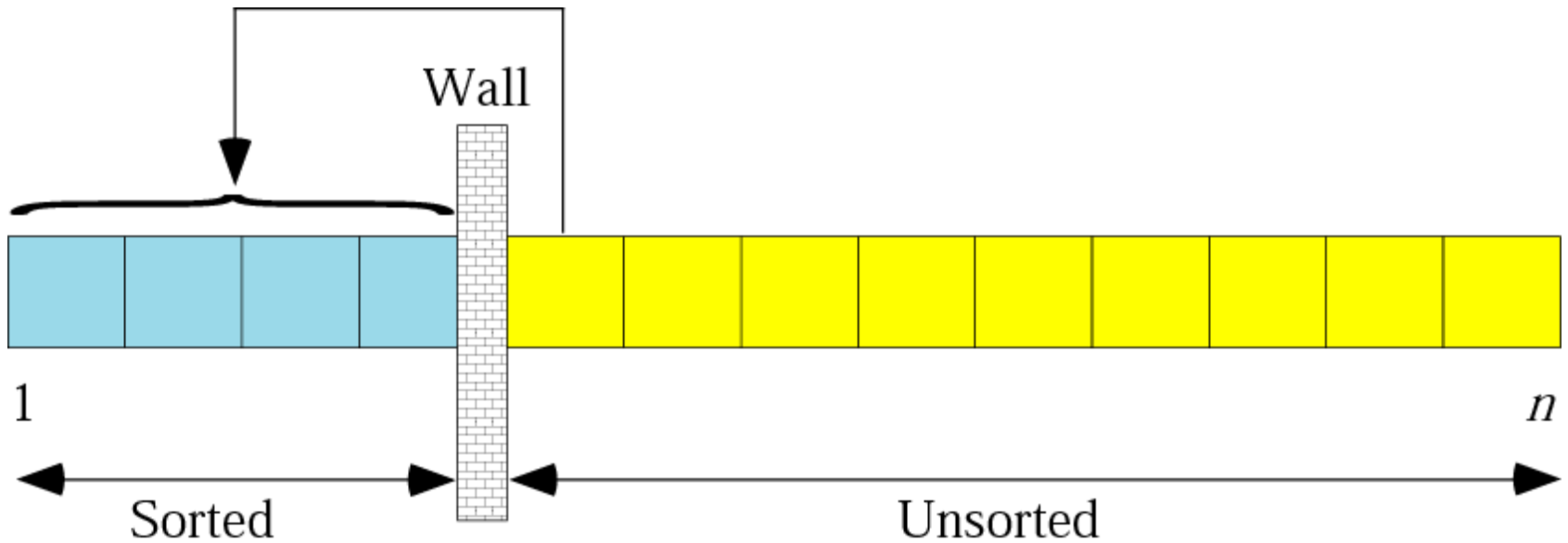


Figure 8-18: part I

Example of insertion sort

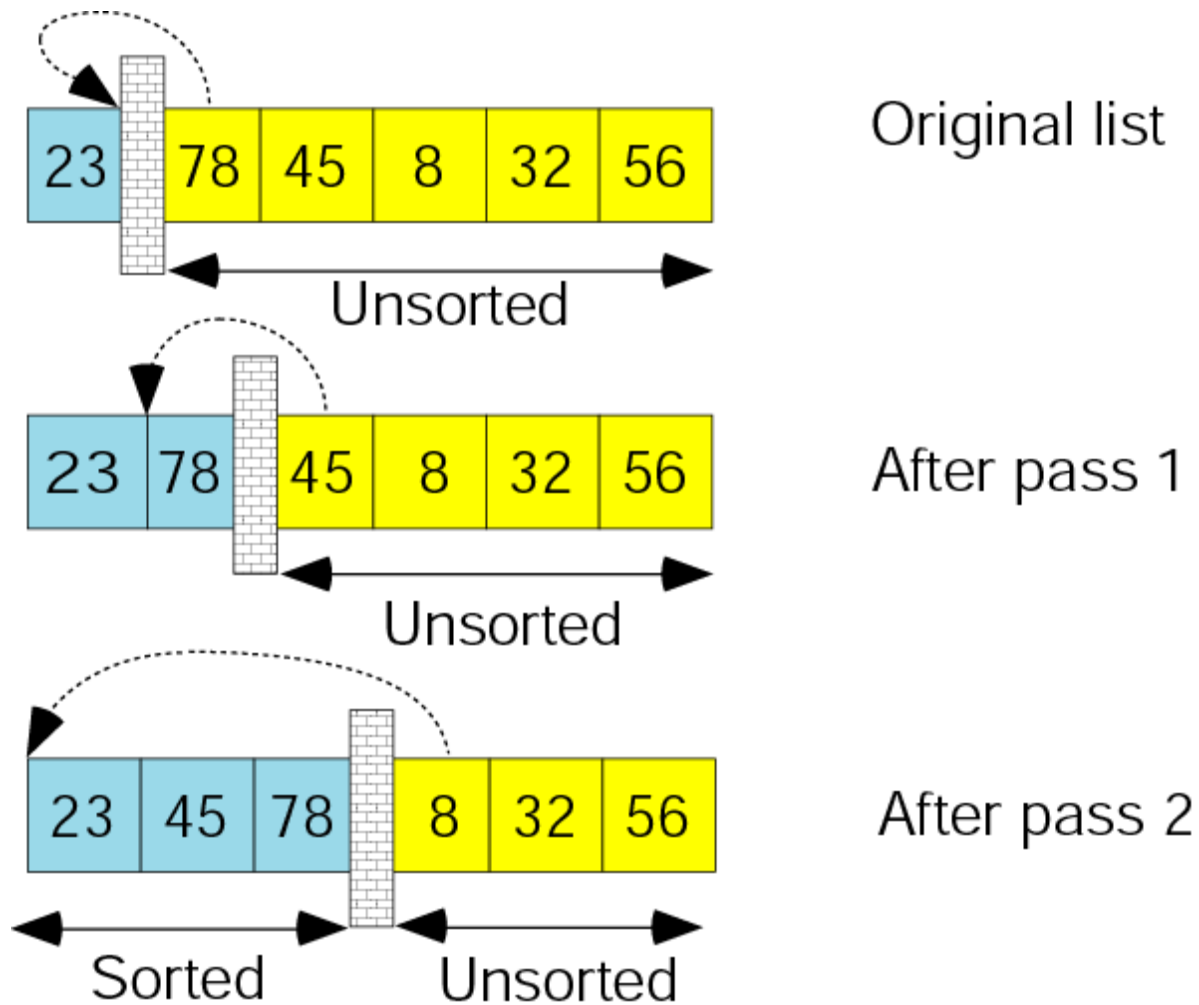
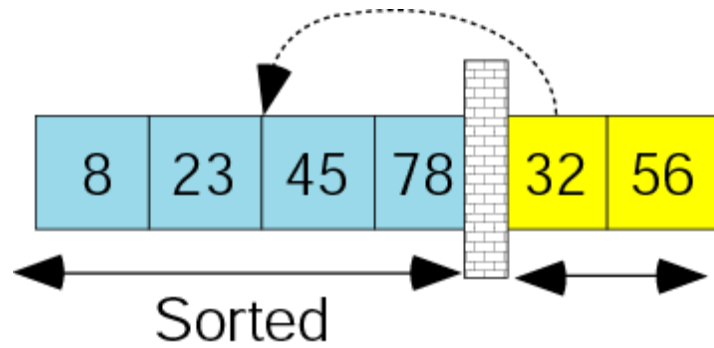
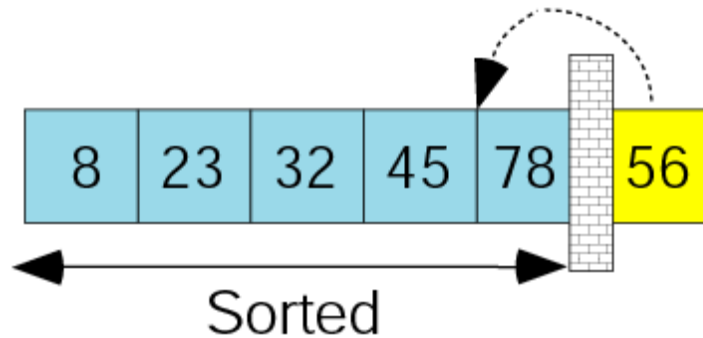


Figure 8-18: part II

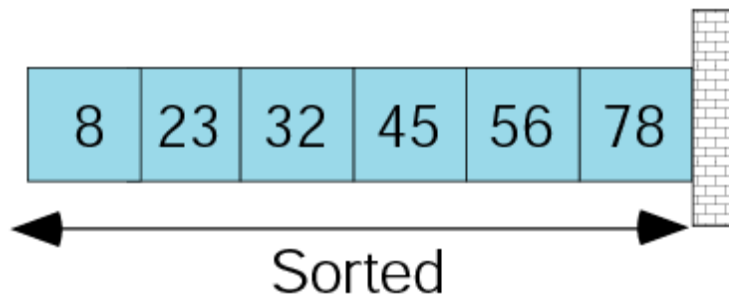
Example of insertion sort



After pass 3



After pass 4



After pass 5

Figure 8-19

Search concept

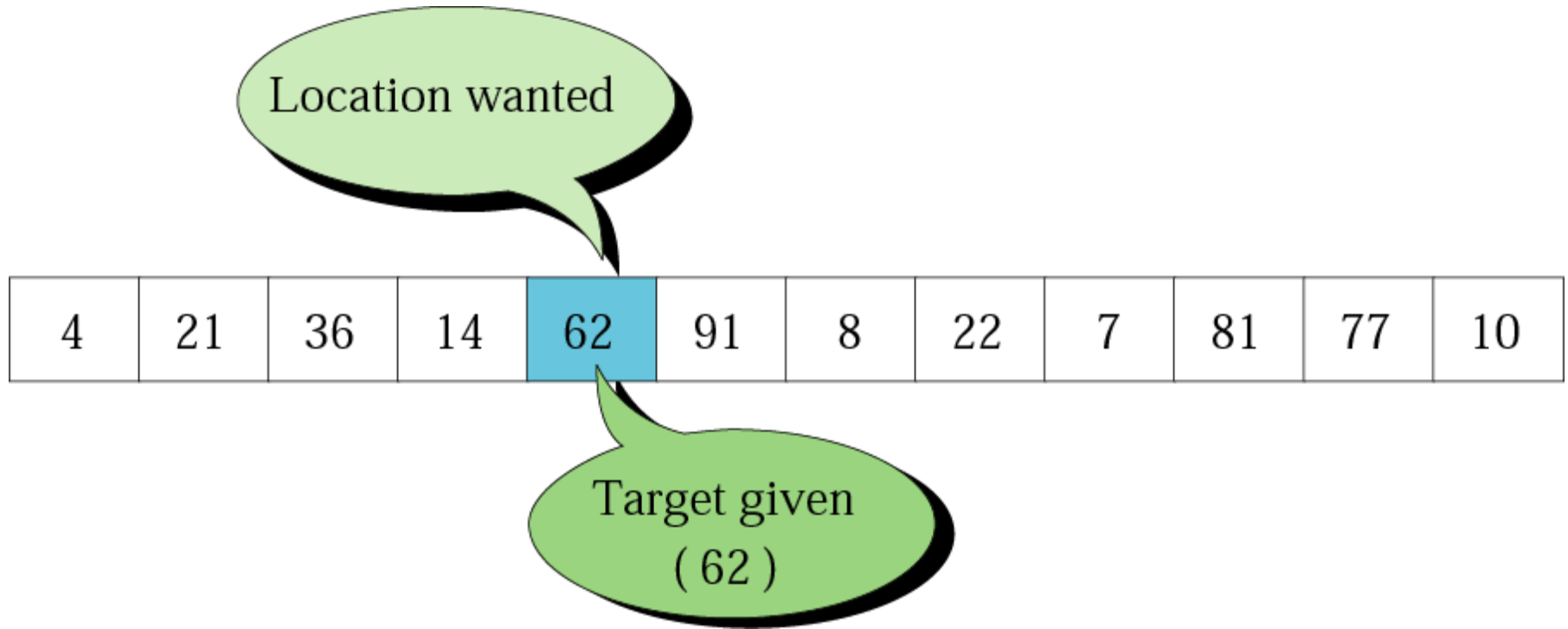


Figure 8-20: Part I

Example of a sequential sort

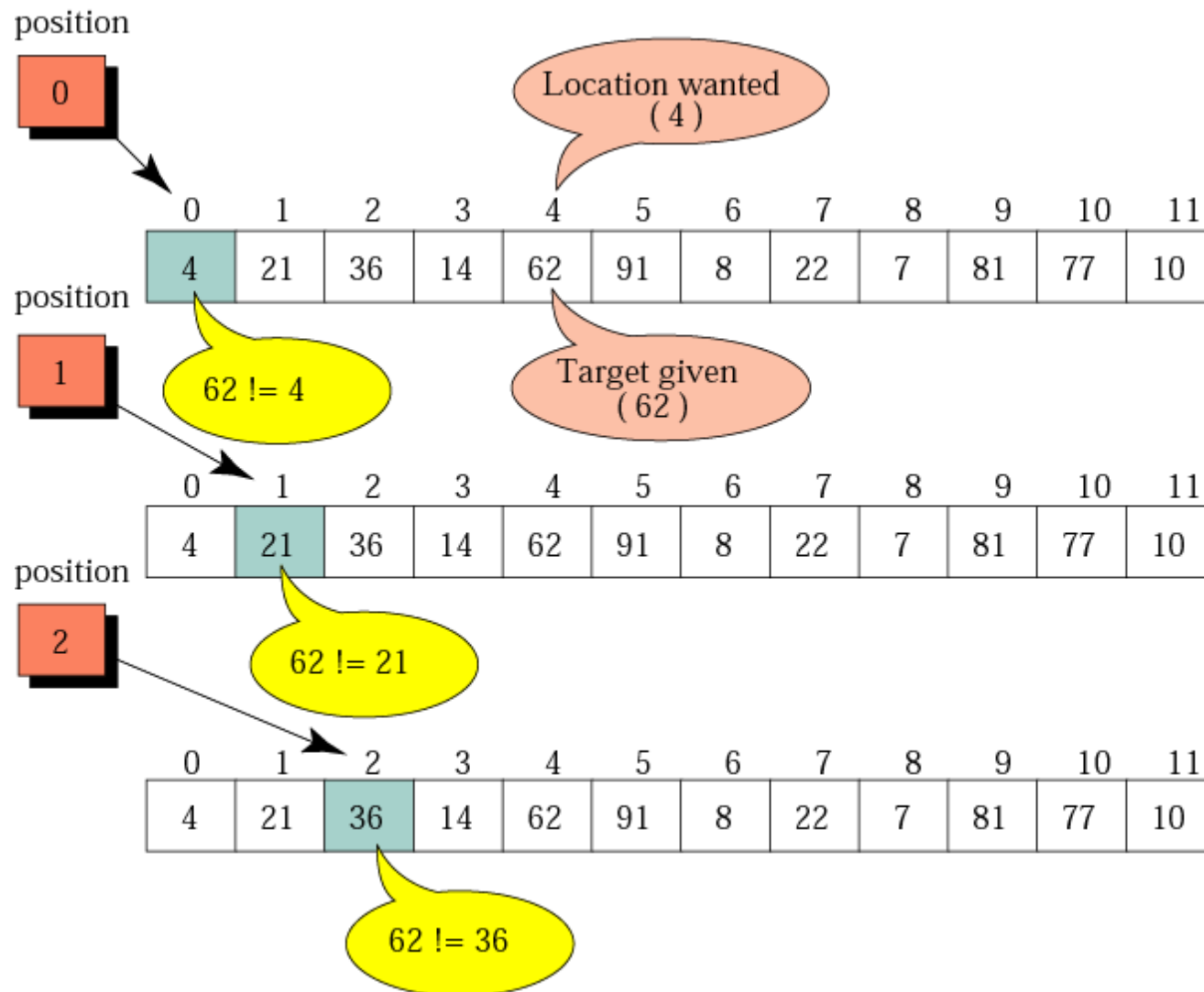


Figure 8-20: Part II

Example of a sequential sort

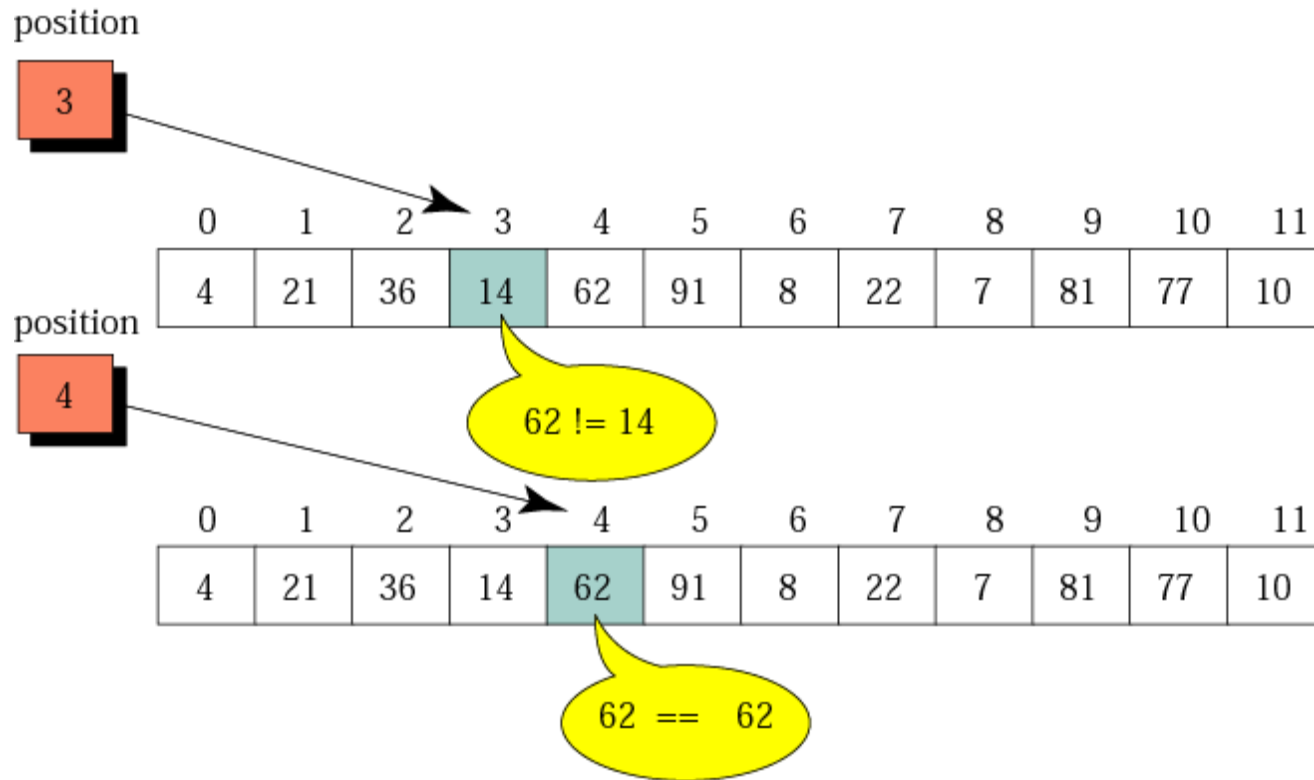
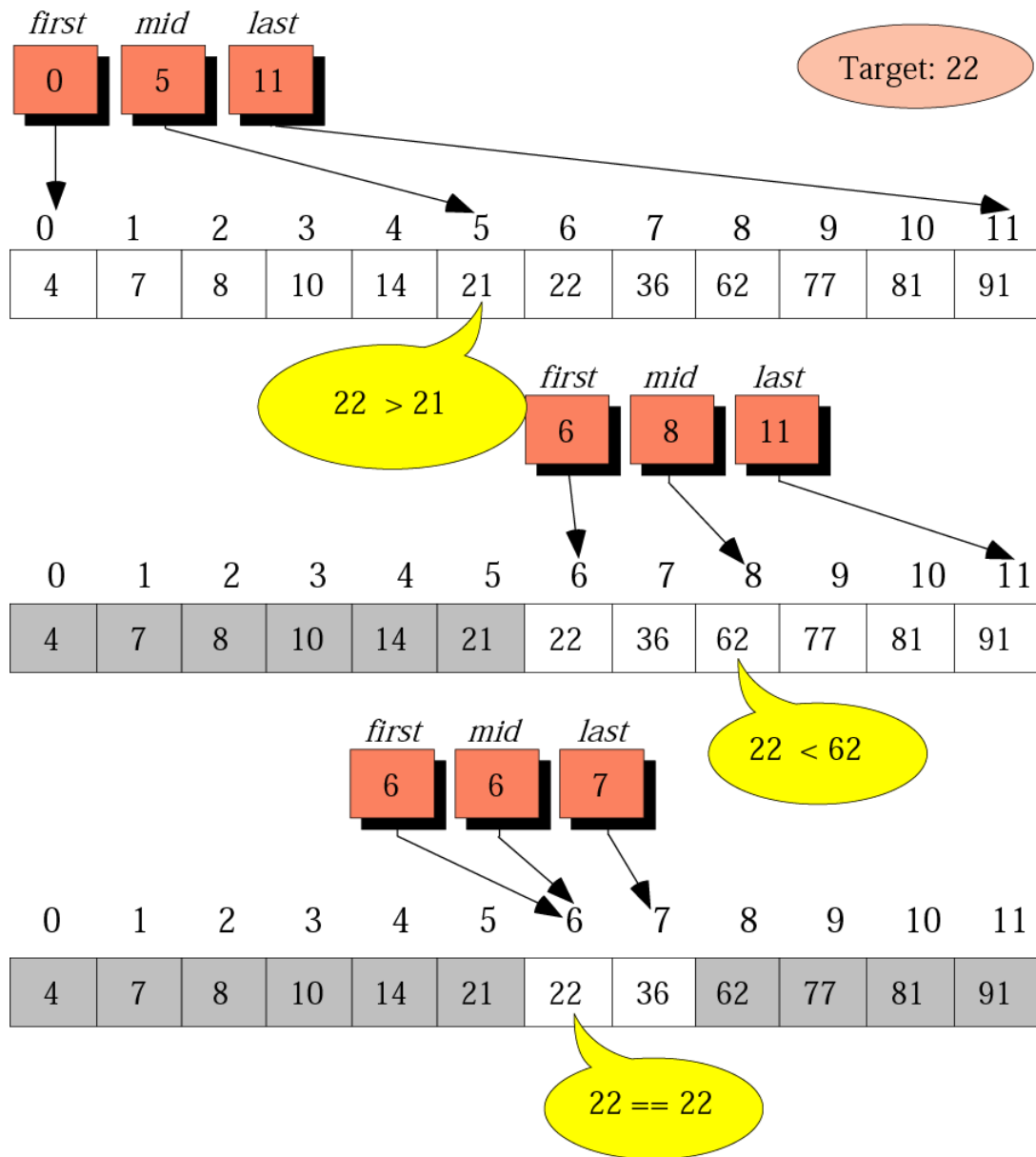


Figure 8-21

Example of a binary sort



8.1

RECURSION

Figure 8-22

Iterative definition of factorial

$$\text{Factorial } (n) = \left[\begin{array}{ll} 1 & \text{if } n = 0 \\ n \times (n-1) \times (n-2) \times \dots \times 3 \times 2 \times 1 & \text{if } n > 0 \end{array} \right]$$

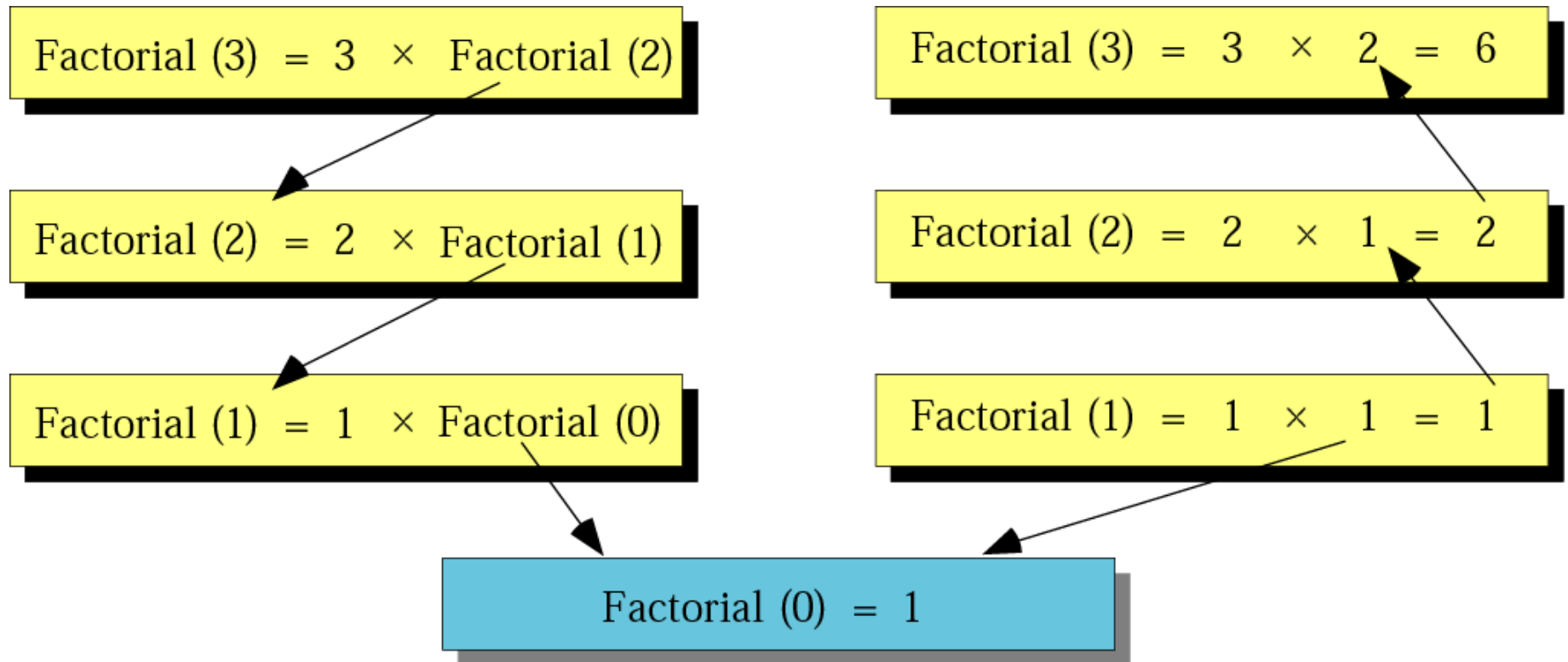
Figure 8-23

Recursive definition of factorial

$$\text{Factorial } (n) = \left[\begin{array}{ll} 1 & \text{if } n = 0 \\ n \times \text{Factorial } (n - 1) & \text{if } n > 0 \end{array} \right]$$

Figure 8-24

Tracing recursive solution to factorial problem



Algorithm 8.7: Iterative factorial

Factorial

Input: A positive integer num

- 1. Set FactN to 1**
- 2. Set i to 1**
- 3. while (i is less than or equal to num)**
 - 3.1 Set FactN to FactN x I**
 - 3.2 Increment i**
- End while**
- 4. Return FactN**

End

Algorithm 8.8: Recursive factorial

Factorial

Input: A positive integer num

- 1. if (num is equal to 0)**
then
 - 1.1 return 1**
 - else**
 - 1.2 return num x Factorial (num – 1)**
- End if**
- End**